EB-05 Beck® Equilibrium Crane

Introduction

Statics is a vital part of the curriculum for science and engineering students. It is at the center of structural engineering. This experiment in static equilibrium uses the practical model of a crane Equilibrium involves the resolution of a number of non-collinear forces. It illustrates as well, the functioning of a most useful construction aid, the crane.

As any engineer should know that, a rigid body can be in static equilibrium *if and only if* the following conditions are satisfied:

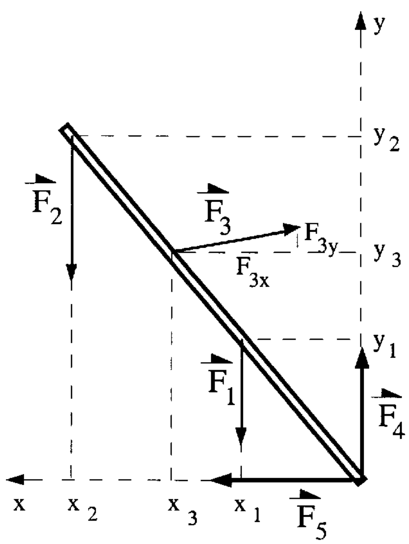
1. The resultant external force (the vector sum of all external forces) acting on the body must be zero, and

Figure One

2. The resultant external torque (the vector sum of all external torques) acting on the body must be zero *about any origin.* These conditions can be written as equations:

$$\sum \mathbf{F} = 0 \qquad \sum \tau = 0$$

where the **F** are the force vector and **τ** are the torque vectors in the apparatus. In this experiment, you will apply forces at different points on a rigid body so that it is in static equilibrium and will then attempt to measure those forces and the positions at which they are applied and to determine or estimate the errors in your measurements. Your measurements will then be used to test whether the above conditions are satisfied. More than three forces will be involved, so their lines of action need not intersect at a point. The rigid body in static equilibrium will be the boom of a crane as pictured in Figure One. The forces acting on it will be its own weight $\mathbf{F}_1$ the weight of a load $\mathbf{F}_2$ hanging vertically from a point on the boom, the tension $\mathbf{F}_3$ (which may have both horizontal and vertical components) in a string supporting the boom, and forces $\mathbf{F}_4$ and $\mathbf{F}_5$ applied near the base of the boom which can be adjusted so that the boom is held in static equilibrium without touching the frame. These forces and their lines of action are schematically shown in the sketch below.



Force $\mathbf{F}_3$ is provided by a dynamometer connected to the boom by a string that passes under a pulley. A scale shows the elongation of the spring. Forces $\mathbf{F}_4$ and $\mathbf{F}_5$ are provided by two dynamometers connected by strings to a frame on the boom near the base. Cords are arranged so $\mathbf{F}_4$ is vertical and $\mathbf{F}_5$ is horizontal and so that their lines of action pass through a common point, about which the boom is pivoted when it is in the proper position. The forces should all be coplanar (with the boom in the same plane) so that their torques about any point in the plane will all be perpendicular to the plane.

Figure Two

Then the two vector equations of equilibrium are equivalent to three scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau_z = 0$$

where the x and y axes are assumed to be horizontal and vertical axes, respectively, in the plane of the forces and the z axis is perpendicular to this plane. Note that, since the sum of the torques about any point (including any point in the xy plane) must be zero, different points may be chosen to eliminate contributions due to some of the forces. Thus, this equation can be used to obtain more than one relation among the quantities that you will measure.

The forces and distances in this experiment may not be simple to determine accurately. You are expected to devise your own methods for making the measurements and for determining the errors in the measured quantities using the equipment provided. A few hints follow:

If one determines the x and y coordinates of several points relative to a common origin, it is then straightforward to determine their positions relative to each other. One can determine the x and y components of a force transmitted by a light string by determining the magnitude of the force and the lengths of the sides of a right triangle with the string forming the hypotenuse and the other sides parallel to the x and y axes. If all three sides of a right triangle are measured, the Pythagorean theorem can be used to provide a check on whether the measurements are consistent. When a force is transmitted from weights by a string passing over pulleys, friction in the pulleys can increase the uncertainty in your determination of the force. Upper and lower limits on the force can often be found by pushing up or pulling down slightly on the weights as the last step in preparation for the measurement. If all forces are measured in terms of the masses of weights, the common factor $\mathbf{g}$ in $\mathbf{F} = m\mathbf{g}$ can be canceled out in many of your calculations to simplify them (but be aware that the results will then not be in proper SI units).

Required Equipment.

Beck Equilibrium Crane Apparatus, weight hanger, weights, meter sticks (2), tape measure, ruler, string.

Procedure

BE CAREFUL NOT TO LET WEIGHTS FALL! You will need to make measurements sufficient to determine *all of the forces* acting on the boom and *all of the resultant torques* about points in the plane of the forces and boom. *In* **all** *of the measurements you are expected to determine errors for* **all** *of the measured quantities,* either by estimation or by repeated measurements or by a combination of these techniques. The equilibrium crane apparatus should be set up approximately as shown in Figure One.

1. Remove the dynamometers and unhook the strings where possible, being careful to remember how they are hooked up. Remove the boom. Determine the location of the center of mass of the boom by balancing it on the edge of a ruler and mark its position. Measure the distance of the center of mass from the pivot of the boom. Weigh the boom on the laboratory balance. Measure the distance from the pivot point to the pins on the boom where strings can be hooked. Draw a sketch of the boom and record your measurements and error estimates.

2. Replace the boom on the frame. Reconnect the $\mathbf{F}_3$ string, via the pulley, to one set of pins on the boom. Reposition the $\mathbf{F}_4$ and $\mathbf{F}_5$ strings and

connect them to their dynamometers. Hang a weight hanger with 300 to 500 grams of additional weights from a string connected to a different set of pins on the boom. Note which pins are used to attach the strings. The bar forming the frame of the crane must be vertical so that you can use it as a reference for measuring horizontal distances and so that the forces are really coplanar with $\mathbf{F}_4$ vertical and $\mathbf{F}_5$ horizontal. A level is available to check this in each room. You should also check to see that the horizontal distances from the bar to the top and bottom of the vertical string providing $\mathbf{F}_2$ are the same. (If adjustment is needed, get the instructor or assistant to help.)

3. Now change the tension on the dynamometers by twisting their mounting screws supplying $\mathbf{F}_4$ and $\mathbf{F}_5$ so that the boom is balanced with the rod through the pivot point *suspended in the* CENTER *of the holes* on the frame where it initially rested.

Since it is difficult to achieve a perfect stable equilibrium where the base of the boom does not tend to move sideways, you will probably have to push lightly sideways at the base and/or the far end of the boom with a pencil or some similar object to keep the boom centered and away from the sides of the support. Try not to add any component of force in the plane of the other forces, since this will not be taken into account in your calculations.

4. Record the weights and errors for $\mathbf{F}_2$, $\mathbf{F}_4$ and $\mathbf{F}_5$ when the boom is in equilibrium in this manner. For $\mathbf{F}_4$ and $\mathbf{F}_5$, make error estimates by adding or subtracting small amounts of weights and lifting up or pushing down on the boom to see how much change is required before a change in boom position can be observed.

5. While the boom is suspended, carefully determine the x and y coordinates on the boom of the points of application of all the forces. (You may also want to determine the separation of the various points as a check.) Carefully record the extension of the spring supplying $\mathbf{F}_3$ and determine the direction of $\mathbf{F}_3$ by measuring the x and y coordinates of two points on the string (and the separation of the points) between the pulley and the boom. Make error estimates for all your measurements.

Check to make sure you have all the measurements necessary. You may want to repeat the process to see how closely the measurements agree on a second try. After you are done, remove the weights but reassemble the system so the next group can see how it fits together.

Analysis

Note that, in the scalar equations on p. 2, the components of vectors may be positive or negative. However, in the equations of this section the magnitudes of the components of forces and displacements have been used so that the signs of terms could be shown corresponding to the situation in Figure Two. In the following calculations and discussion you may express forces in terms of masses for equivalent weights, even though the units will not be those of force.

In terms of the forces on the boom, the first two of the scalar equations for equilibrium are:

$$F_5 - F_x = 0$$

$$F_4 + F_{3y} - F_2 - F_1 = 0$$

Use your measurements to determine the values and errors for the quantities on the left sides of these equations. Compute the x and y components and magnitude (and errors) for the net force on the boom as determined from your measurements. (Make sure that the signs of the terms are correct for your setup.)

On a sheet of graph paper, perform a careful graphical addition of forces $\mathbf{F}_1$ through $\mathbf{F}_5$. Determine the magnitude and the x and y components of any net resultant force on the boom from your graph. How do these results compare with the analytical calculations?

As a check on your measurements of the coordinates for the application points of the forces, plot these points, with error bars, on a sheet of graph paper. They were in a straight line along the boom. Are they in your plot? Draw a "best" straight line through the points. Also check the distances between points against your measurements. Depending on your results, you may want to adjust the distances used in the following torque calculations a bit, but be careful to be consistent and to explain what you do. Do NOT make adjustments just to make the resultant torque look small.

If torques are calculated with respect to the pivot point on the boom, there should be no contribution from $\mathbf{F}_4$ and $\mathbf{F}_5$. The resulting scalar equation is (with CLOCKWISE torques taken to be positive to maintain a right-handed coordinate system since the +x axis is drawn to the left):

$$F_{3x}\, y_3 + F_{3y}\, x_3 - F_2\, x_2 - F_1 x_1 = 0$$

Compute the various terms in this equation and their errors. Compute the net torque about the axis and its error.

Write the corresponding scalar equation for *torques about* the application point of $\mathbf{F}_3$ on the boom, making sure that the signs of the terms correspond to the situation for your measurements. Compute the net torque (and its error) about this axis. (Note that this does not depend on your $\mathbf{F}_3$.)

Do the same thing for axes at the application points of $\mathbf{F}_2$ and $\mathbf{F}_1$.

Now pick an arbitrary point in the plane of the forces that is neither on the boom nor on the lines of action of any of the forces and repeat the process. Note that each of these equations involves different combinations of the forces and distances you measured.

Discussion of Results

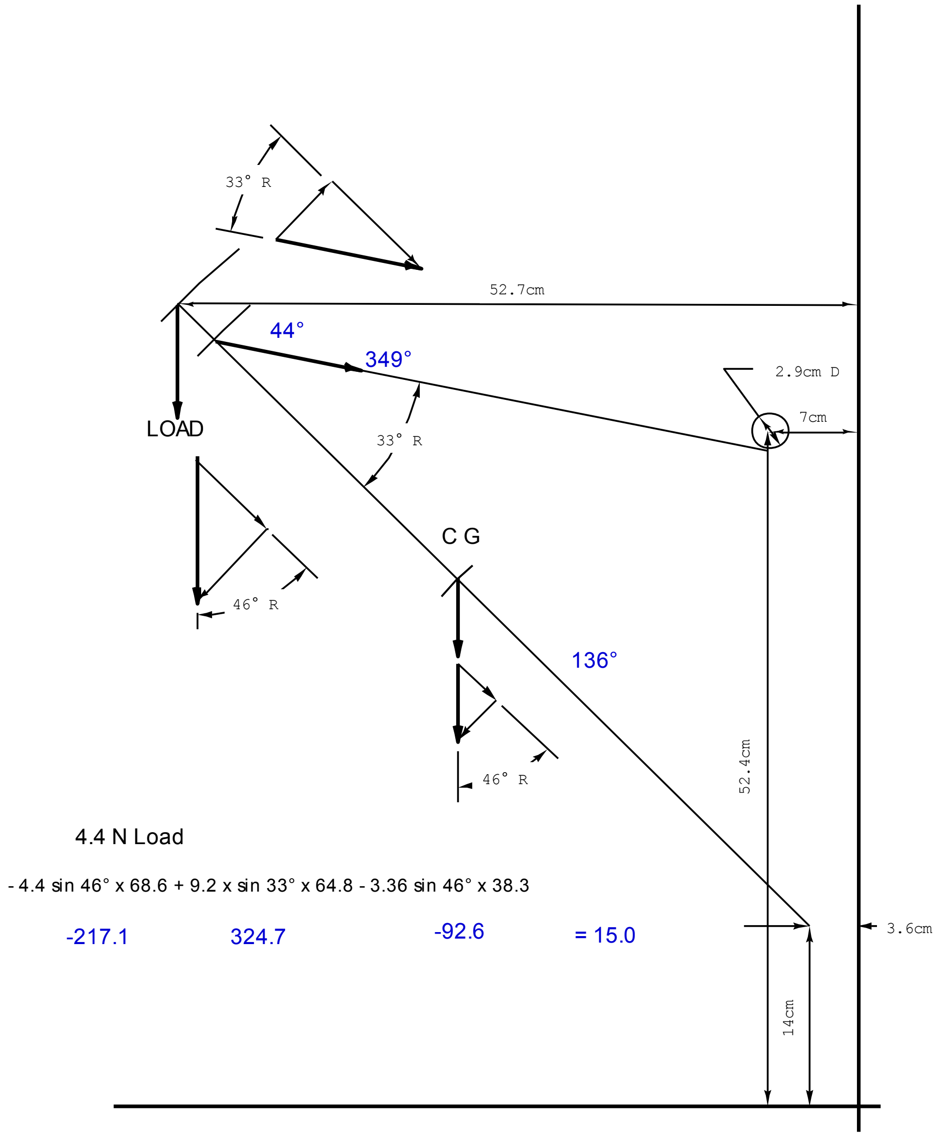
Discuss your results. Do you conclude that they are consistent with the net force on a rigid body in static equilibrium being zero? Are they consistent with the net torque about an arbitrary axis being zero for a rigid body in static equilibrium? If yes, tell how you have reached these conclusions. If no, what do you think could be happening? Do your results indicate that your determinations of the errors in the measurements are reasonable? Too large or too small? Discuss the ways in which you made your error measurements. Are there indications from the scalar equations that certain measurements and/or error determinations are less (or more) accurate than others? What evidence is there for this? Can you think of ways to improve the measurements and error determinations if the experiment were to be repeated? Can you think of ways in which the apparatus might be improved in order to make the measurements easier or more accurate? What are the proper SI units for force and torque? What changes would have to be made in your calculations to express your results in proper SI units?

NOTE: Before you leave the laboratory make sure you have made all the required measurements and estimates of errors needed to complete the experiment. You must write out the appropriate torque equation for at least one other axis besides the pivot point. If time allows, you should check to see that your results are at least roughly consistent by substituting values into some of the scalar equations.

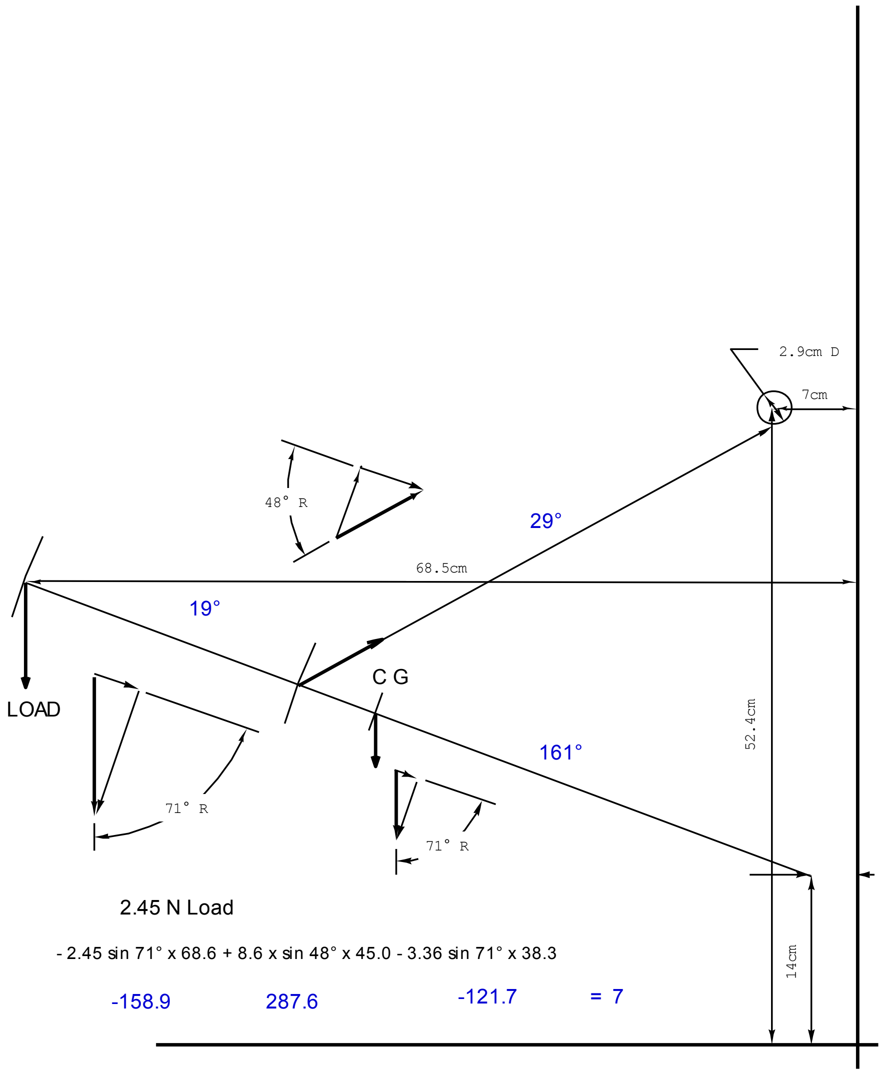
This manual is based upon the manual used at Notre Dame University written by Mike McFarland in the Physics Department. His e-mail address is: mmcfarla@iron.helios.nd.edu.

Typical Configurations

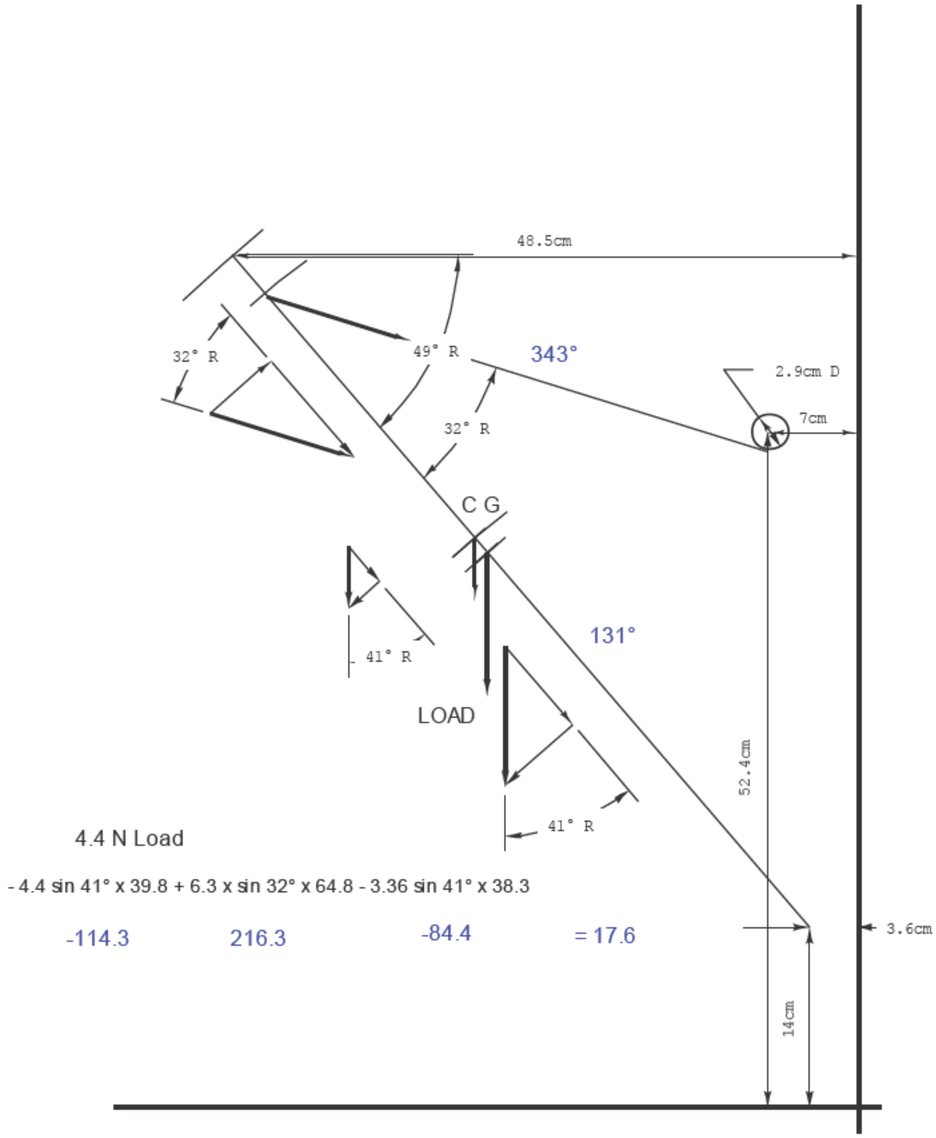
This figure illustrates a typical arrangement of forces and a calculation of the sum of the moments. None of the measurements will likely match your data. The results should be similar to the values you have obtained. There are a variety of configurations available by changing the load and support points. Four of these variants are shown in the following figures.



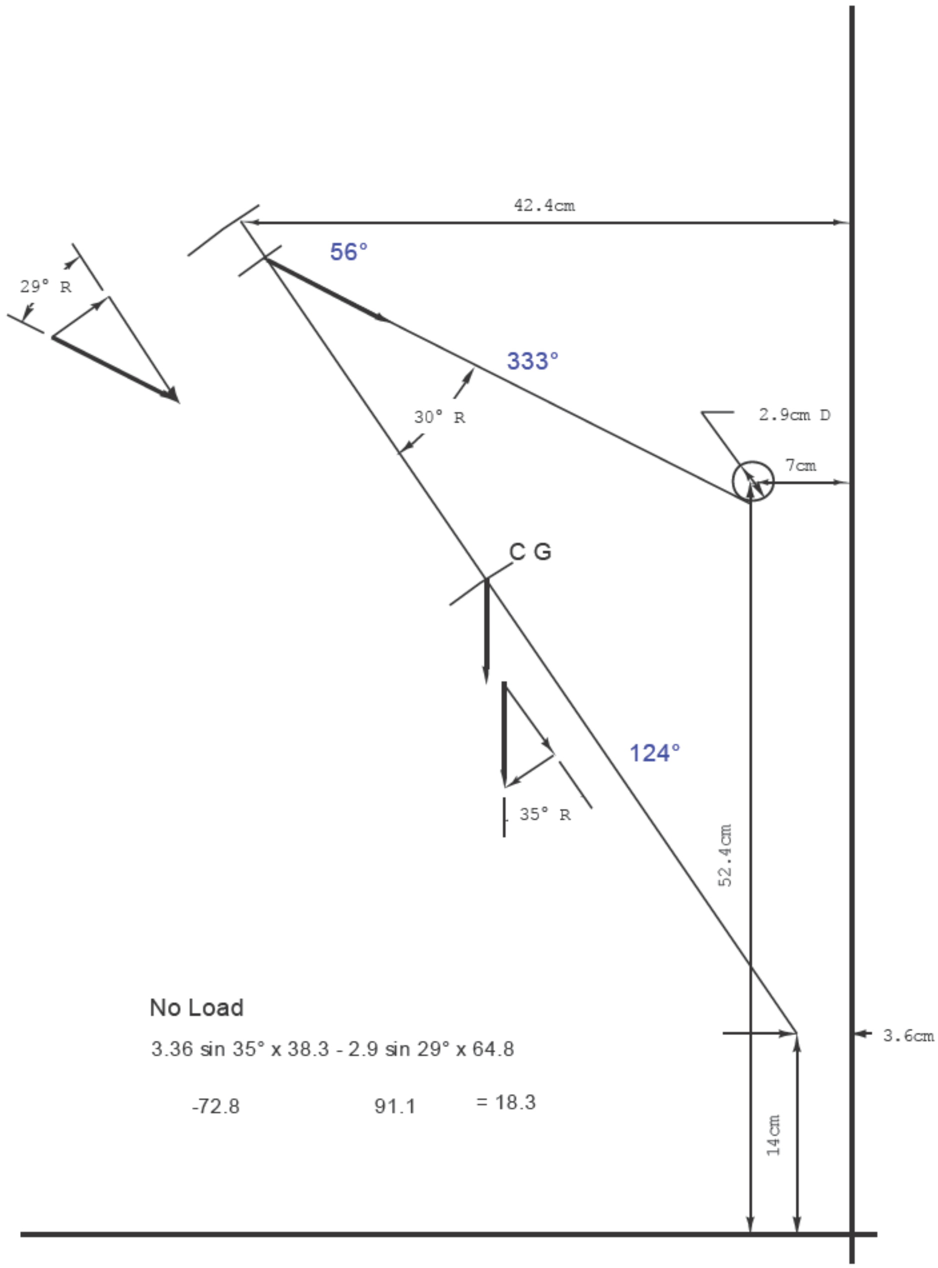
This might be considered the normal configuration. The load is suspended near the end of the boom and the boom support line is also fastened near the end.



This configuration has the boom nearly horizontal which would be required if the load to be picked up was a long way from the column of the crane.



This configuration supports the load from near the middle of the boom, close to but not coincident with the center of gravity of the boom.



No Load

3.36 sin 35° x 38.3 - 2.9 sin 29° x 64.8

-72.8 91.1 = 18.3

This configuration has no load attached to the crane. All the support string carries is the weight of the boom so there are only two moments to sum.

WRJB-December 2004

Warranty and Parts: We replace all defective or missing parts free of charge. We accept MasterCard, Visa, checks and School P.O.s. All products warranted to be free from defect for 90 days. Does not apply to accident, misuse or normal wear and tear. Intended for children 13 years of age and up. This item is not a toy. It may contain small parts that can be choking hazards. Adult supervision is required.